**Math 233** **Name (Print):** ____________________

**Spring 2017**

**Quiz 8**

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1. (40 points) **Linearly Independent Eigenvectors:** Suppose that $\{v_1,\ v_2\}$ is a set of linearly independent eigenvectors of $A$ with corresponding eigenvalues $\lambda_1,\ \lambda_2$.

   (a) (10 points) Let $v_3$ be a third eigenvector with eigenvalue $\lambda_3 \neq \lambda_i$ for $i = 1, 2$. What must be true if the vectors $\{v_1,\ v_2,\ v_3\}$ are linearly dependent.

   (b) (10 points) Assume that $v_3 = c_1 v_1 + c_2 v_2$. Apply $A$ to both sides of this equation to get a new expression for $v_3$.

   (c) (10 points) Take the two expressions from part $(b)$, explain why the coefficients on $v_1$ and $v_2$ must be the same in both expression.

   (d) (10 points) Show that $c_1$ and $c_2$ must be zero, thus providing us with a contradiction since $v_3 = 0$ isn't an eigenvector.

2. (30 points) Consider the matrix

$$A = \begin{bmatrix} 5 & 0 & -1 \\ 0 & 2 & 0 \\ 6 & 0 & 0 \end{bmatrix}$$

(a) (10 points) Find the characteristic polynomial, eigenvalues, and eigenvectors for this matrix.

(b) (10 points) For each eigenvalue, list the geometric and algebraic multiplicity.

(c) (10 points) Verify that the eigenvalues are linearly independent and thus form a basis for $\mathbb{R}^3$.

3. (30 points) The matrix $A = \begin{bmatrix} 5 & -1 \\ 4 & 1 \end{bmatrix}$ has a double root in its characteristic polynomial, but only one eigenvector.

   (a) (15 points) Find the characteristic polynomial, the eigenvalue, and the eigenvector.

   (b) (15 points) When we don't have enough eigenvectors to form a basis, there's a generalization of the eigenvalue problem. Eigenvectors satisfy the equation $(A-\lambda I)v = 0$, generalized eigenvectors satisfy the equation $(A - \lambda I)^n v = 0$. Show that $(A - \lambda I)^2$ has a two dimensional nullspace. Create an orthonormal basis for $\mathbb{R}^2$ consisting of one eigenvector and one generalized eigenvector.