Math 222
Spring 2017
Quiz 5

Name (Print): _____

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1. (45 points) Prove whether the following series converge or diverge using the suggested test. If they converge, calculate the final sum using Mathematica (decimal answers, not exact).

   (a) (15 points) **P-Series Test:** $\displaystyle\sum_{n=1}^{\infty} \frac{4}{\sqrt{n^3}}$

   (b) (15 points) **Integral Test:** $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n+5}$

   (c) (15 points) **Comparison Test:** $\displaystyle\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^{n^2}$

2. (55 points) **Telescoping Series:** The comparison test can be used to show the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ coverages. This series can also be calculated exactly by recognizing the telescoping pattern between consecutive terms.

   (a) (10 points) Show that $\frac{1}{n} - \frac{1}{n+1} = \frac{1}{n^2+n}$. This is the partial fraction decomposition.

   (b) (5 points) Write out the first 5 terms of the series using this new form to figure out the pattern.

   (c) (10 points) Find an explicit form for the partial sums $\{S_n\}$.

   (d) (10 points) Find the infinite sum.

   (e) (20 points) Repeat steps (a) through (d) to calculate the infinite sum $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.

   (Hint: The cancelations don't start till the third term of the series.)