Math 222
Spring 2017
Exam 3

Name (Print): _____

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1. (27 points) Proficiency Check: Calculate the following (please show all work)...

   (a) (9 points) $\int x^3 \ln(x)\, dx$

   (b) (9 points) $\int e^x \cos(x)\, dx$

   (c) (9 points) $\int (1-x)e^{x/2}\, dx$

2. (28 points) Suppose the first couple terms of the Taylor series for $f(x)$ and $g(x)$ centered at $x_0 = 0$ are

$$f(x) = 7 - 6x + 5x^2 + \dots$$
$$g(x) = 2 + 3x + 4x^2 + \dots$$

(a) (5 points) Find $f(0)$, $f'(0)$, $f''(0)$, $g(0)$, $g'(0)$, and $g''(0)$.

(b) (8 points) Find the Taylor series for $s(x) = f(x) + g(x)$ centered at $x_0 = 0$ up to the x^2 term.

(c) (8 points) Find the Taylor series for $p(x) = f(x)g(x)$ centered at $x_0 = 0$ up to the x^2 term.

(d) (7 points) **Explain** why we can't find the x^3 term in the Taylor series for $p(x)$.

3. (24 points) Find the radius and interval of convergence for the following Power series. Cite any convergence tests you may use and clearly state how you are using them.

   (a) (12 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(x-3)^n}{5^n}$

   (b) (12 points) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1} x^n$

4. (21 points) Consider the function defined by the series

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{x^n}$$

(a) (5 points) Calculate $f(2)$.

(b) (8 points) Use the ratio test to find where this function is well defined.

(c) (8 points) Use the Famous Taylor series to find the function that's represented by this series wherever the series converges, i.e., $f(x)$.